ANSWER KEY

1.	B
2.	B
3.	A
4.	A
5.	A
6.	B
7.	C
8.	B
9.	C
10.	B
11.	D
12.	B
13.	B
14.	D
15.	C
16.	C
17.	A
18.	B
19.	B
20.	A
21.	B
22.	A
23.	C
24.	C
25.	A
26.	C
27.	A
28.	B
29.	C
30.	D

SOLUTION

1. $S_n = 2n^2 + 5n$

$\therefore S_{n-1} = 2(n-1)^2 + 5(n-1)$

$\therefore T_n = S_n - S_{n-1}$

$= 2[2n - 1] + 5 = 4n + 3$

2. $\dfrac{S_n}{S'_n} = \dfrac{3n+8}{7n+15}$

$\Rightarrow \dfrac{\frac{n}{2}[2a_1 + (n-1)d_1]}{\frac{n}{2}[2a_2 + (n-1)d_2]} = \dfrac{3n+8}{7n+15}$

$\Rightarrow \dfrac{a_1 + \left(\frac{n-1}{2}\right)d_1}{a_2 + \left(\frac{n-1}{2}\right)d_2} = \dfrac{3n+8}{7n+15}$

put $\dfrac{n-1}{2} = 11 \therefore n = 23$

$\Rightarrow \dfrac{a_1 + 11d_1}{a_2 + 11d_2} = \dfrac{3.23+8}{7.23+15} = \dfrac{77}{176} = \dfrac{7}{16}$

3. $\dfrac{S_n}{S'_n} = \dfrac{2n}{n+1}$

To find $\dfrac{T_N}{T'_N}$

Simply put $n = 2N - 1$

$\because N = 8 \therefore n = 15$

$\therefore \dfrac{T_8}{T'_8} = \dfrac{2.15}{15+1} = \dfrac{30}{16} = \dfrac{15}{8}$

4. $\because a^2(b+c),\ b^2(c+a),\ c^2(a+b)$: AP

* Dividing by abc

$\therefore \dfrac{ab+ac}{bc}, \dfrac{bc+ab}{ac}, \dfrac{ac+bc}{ab}$: AP

* Adding 1 in all & dividing by $(ab + bc + ca)$

$\dfrac{1}{bc}, \dfrac{1}{ac}, \dfrac{1}{ab}$: AP

Multiplying by abc

a, b, c : AP

5. Given a, b, c are in AP

Now $\dfrac{1}{\sqrt{b}+\sqrt{c}}, \dfrac{1}{\sqrt{c}+\sqrt{a}}, \dfrac{1}{\sqrt{a}+\sqrt{b}}$: ?

Rationalising

$\Rightarrow \dfrac{\sqrt{c}-\sqrt{b}}{c-b}, \dfrac{\sqrt{c}-\sqrt{a}}{c-a}, \dfrac{\sqrt{b}-\sqrt{a}}{b-a}$: ?

$\Rightarrow \dfrac{\sqrt{c}-\sqrt{b}}{d}, \dfrac{\sqrt{c}-\sqrt{a}}{2d}, \dfrac{\sqrt{b}-\sqrt{a}}{d}$: ?

Clearly $T_2 = \dfrac{T_1 + T_3}{2}$

$\therefore$ in AP

6. Let no. of terms is 2n

$\therefore$ GP is $a, ar, ar^2, ar^3 \ldots\ldots, ar^{2n-2}, ar^{2n-1}$

Now $S_{all} = 3\,S_{odd}$

$\Rightarrow a\left(\dfrac{r^{2n}-1}{r-1}\right) = 3.a\left(\dfrac{r^{2n}-1}{r^2-1}\right)$

$\Rightarrow \dfrac{1}{r-1} = \dfrac{3}{r^2-1}$

$\therefore r = 2$

7. $a = 1$

$\therefore \quad S = \dfrac{a}{1-r} = \dfrac{1}{1-r}$

$\therefore \quad r = 1 - \dfrac{1}{S} = \dfrac{S-1}{S}$

Now sum of squares of GP

$= \dfrac{a^2}{1-r^2} = \dfrac{1}{1-\left(\dfrac{S-1}{S}\right)^2}$

$= \dfrac{S^2}{S^2-(S-1)^2} = \dfrac{S^2}{2S-1}$

8. GP : a, ar, ar^2

$\because \quad \boxed{a + ar + ar^2 = 65}$ (i)

& $a.ar.ar^2 = (ar)^3 = 3375$

$\therefore \quad \boxed{ar = 15}$

putting $r = \dfrac{15}{a}$ in (i)

we get $\boxed{a = 5}$

9. Let numbers are a & b

$\because \quad \dfrac{a+b}{2} = 34 \Rightarrow \boxed{a+b=68}$ ...(i)

& $\sqrt{ab} = 16 \Rightarrow ab = 256$

$\because \quad (a-b)^2 = (a+b)^2 - 4ab$

$(a-b)^2 = (68)^2 - 4.256$

$\therefore \quad \boxed{a-b=60}$ ...(ii)

Solving (i) & (ii) $\left.\begin{array}{l} a = 64 \\ b = 4 \end{array}\right]$

10. $\because \quad \boxed{a = 4b}$ (given) (i)

If A – G = 2

$\therefore \quad \dfrac{a+b}{2} - \sqrt{ab} = 2$

$\therefore \quad a + b - 2\sqrt{ab} = 4$

$\therefore \quad (\sqrt{a} - \sqrt{b})^2 = 4$

Using (i)

$\Rightarrow \quad (2\sqrt{b} - \sqrt{b})^2 = 4$

$\Rightarrow \quad b = 4$

$\therefore \quad a = 16$

11. Here in AP; a = 1, d = 1

in GP x = 1, $r = 1 + \dfrac{1}{n}$

$\therefore \quad S_\infty = \dfrac{ax}{1-r} + \dfrac{rdx}{(1-r)^2}$

$= \dfrac{1}{1-r} + \dfrac{r}{(1-r)^2}$

$= \dfrac{1}{1-r}\left(1 + \dfrac{r}{1-r}\right)$

$= \dfrac{1}{(1-r)^2} = \dfrac{1}{\left(1-1-\dfrac{1}{n}\right)^2} = n^2$

12. Let $S = \dfrac{2}{3} + \dfrac{6}{3^2} + \dfrac{10}{3^3} + \dfrac{14}{3^4}$ ....

$\therefore \quad \dfrac{S}{3} = \dfrac{2}{3^2} + \dfrac{6}{3^3} + \dfrac{10}{3^4}$ ....

On subtracting

$\dfrac{2S}{3} = \dfrac{2}{3} + \left(\dfrac{4}{3^2} + \dfrac{4}{3^3} + \dfrac{4}{3^4} \ldots\ldots\right)$

$\Rightarrow S = 1 + \dfrac{3}{2} \times \dfrac{4}{3^2}\left[1 + \dfrac{1}{3} + \dfrac{1}{3^2} \ldots\ldots\right]$

$\Rightarrow S = 1 + \dfrac{2}{3}\left[\dfrac{1}{1-\dfrac{1}{3}}\right] = 2$

Sum = 1 + S = 1 + 2 = 3

13. For corresponding AP

$T_4 = \frac{5}{3} \Rightarrow a + 3d = \frac{5}{3}$

& $T_8 = 3 \Rightarrow a + 7d = 3$

Solving $a = \frac{2}{3}$; $d = \frac{1}{3}$

Hence for HP : first term $= \frac{1}{a} = \frac{3}{2}$.

14. Corresponding AP : $\frac{1}{2}, \frac{2}{5}, \frac{3}{10}$ ....

$\therefore \quad a = \frac{1}{2}; d = \frac{2}{5} - \frac{1}{2} = \frac{-1}{10}$

Now $T_5 = a + 4d = \frac{1}{2} - \frac{4}{10} = \frac{1}{10}$

$\therefore$ for HP : fifth term = 10.

15. For corresponding AP:

$$\left.\begin{array}{l} T_1 = \frac{1}{a} \\ T_2 = \frac{1}{b} \end{array}\right] d = \frac{1}{b} - \frac{1}{a}$$

Now $T_n = a + (n-1) d$

$\therefore \quad T_n = \frac{1}{a} + (n-1)\left(\frac{1}{b} - \frac{1}{a}\right)$

$\therefore \quad T_n = \frac{1}{a} + \frac{(a-b)(n-1)}{ab}$

$\therefore \quad T_n = \frac{b+(a-b)(n-1)}{ab}$

$\therefore$ For HP : n^{th} term $= \frac{ab}{b+(a-b)(n-1)}$

16. $A + H = 25$ & $G = 12$

$\because \quad G^2 = AH \Rightarrow 144 = AH \therefore H = \frac{144}{A}$

Using $A + H = 25$

$A + \frac{144}{A} = 25$

$\Rightarrow \quad A^2 - 25A + 144 = 0$

$\therefore \quad A = 9$

But $A > G$ So $A = 9$ is rejected

or $\quad A = 16$

$\Rightarrow \quad \frac{a+b}{2} = 16$

$\therefore \quad a + b = 32$

17. Now $A - G = 5$

$\frac{a+b}{2} - \sqrt{ab} = 5$

$\Rightarrow \quad a + b - 2\sqrt{ab} = 10$

$\Rightarrow \quad (\sqrt{a} - \sqrt{b})^2 = 10$ & $G - H = 4$

$\sqrt{ab} - \frac{2ab}{a+b} = 4$

$\sqrt{ab}\,(a + b - 2\sqrt{ab}) = 4(a + b)$

$\Rightarrow \quad \sqrt{ab}\,(\sqrt{a} - \sqrt{b})^2 = 4(a + b)$

$\Rightarrow \quad 10 = 4\left(\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}}\right)$

Put $t = \sqrt{\frac{a}{b}} \Rightarrow 10 = 4t + \frac{4}{t}$

$\Rightarrow \quad 2t^2 - 5t + 2 = 0$

$\therefore \quad t = 2$ or $\frac{1}{2} \quad \therefore \sqrt{\frac{a}{b}} = 2$

$\therefore \quad \boxed{a = 4b}$

$\because \quad (\sqrt{a} - \sqrt{b})^2 = 10$

$\Rightarrow \quad (2\sqrt{b} - \sqrt{b})^2 = 10$

$\Rightarrow \quad b = 10$

$\therefore \quad a = 40$

18. $\Sigma k^3 = \left[\frac{k(k+1)}{2}\right]^2 = (\Sigma k)^2$.

19. Series is : $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2$ ....

n is even : sum = $\frac{n(n+1)^2}{2}$

n is odd : Let the sum is S

$\therefore$ n + 1 is even :

$$\therefore \text{Sum} = \frac{(n+1)(n+2)^2}{2} = S + \underbrace{2\cdot(n+1)^2}_{(n+1)^{th}\text{ term}}$$

$$\therefore S = \frac{(n+1)(n+2)^2}{2} - 2(n+1)^2$$

$$\therefore S = (n+1)\left[\frac{(n+2)^2}{2} - 2(n+1)\right]$$

$$\therefore S = (n+1)\left[\frac{n^2 + 4n + 4 - 4n - 4}{2}\right]$$

$$\therefore S = \frac{n^2(n+1)}{2}$$

20. 0.7 + 0.77 + 0.777 + ... 20 terms

7 [0.1 + 0.11 + 0.111 + ... 20 terms]

$\frac{7}{9}$ [0.9 + 0.99 + 0.999 + ... 20 terms]

$$\frac{7}{9}\left[\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{100}\right)+\text{.........20 terms}\right]$$

$$\frac{7}{9}\left[20-\left(\frac{1}{10}+\frac{1}{10^2}+\text{.......... 20 terms}\right)\right]$$

$$\frac{7}{9}\left[20-\frac{\frac{1}{10}\left(1-\frac{1}{10^{20}}\right)}{\left(1-\frac{1}{10}\right)}\right]$$

$$\frac{7}{81}\left[180-1+\frac{1}{10^{20}}\right] = \frac{7}{81}(179 + 10^{-20})$$

21. $\because$ S = 1 + 3 + 6 + 10 + 15 + ... + 5050

S = 1 + 3 + 6 + 10 + ... + 5050

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0 = 1 + 2 + 3 + 4 + 5 + ... n^{th} term – 5050

5050 = 1 + 2 + 3 + ... n term

$$\therefore \frac{n(n+1)}{2} = 5050$$

$\Rightarrow$ n(n + 1) = 10100 $\Rightarrow$ n (n + 1) = 100 × 101

$\Rightarrow$ n = 100

Let $S = 1^2 + 2^2x + 3^2x^2$ ....

$\therefore$ $S = 1 + 4x + 9x^2 + 16x^3$ .... ∞

$\therefore$ $xS = x + 4x + 9x^3$ .... ∞

On subtracting

$(1-x)S = 1 + 3x + 5x^2 + 7x^3$ ..... ∞ ...(1)

Let $S' = 1 + 3x + 5x^2 + 7x^3$ ...... ∞

Here in AP $\left[\begin{matrix} a = 1 \\ d = 2\end{matrix}\right.$ & GP $\left[\begin{matrix} x = 1 \\ r = x\end{matrix}\right.$

$$\therefore S' = \frac{a}{1-r} + \frac{rd}{(1-r)^2}$$

$$\therefore S' = \frac{1}{1-x} + \frac{2x}{(1-x)^2} = \frac{1}{1-x}\left(1+\frac{2x}{1-x}\right)$$

$$\therefore S' = \frac{1+x}{(1-x)^2}$$

Putting in (1)

$$(1-x)S = \frac{1+x}{(1-x)^2} \Rightarrow \boxed{S = \frac{1+x}{(1-x)^3}}$$

22. a, b, c are in GP

$$\therefore \frac{b}{a} = \frac{c}{b} \Rightarrow \frac{a+b}{a} = \frac{b+c}{b}$$

But $\frac{a+b}{2} = p$ & $\frac{b+c}{2} = q$

$$\Rightarrow \frac{2p}{a} = \frac{2q}{b} \Rightarrow \boxed{\frac{a}{p} = \frac{b}{q}}$$

Now, $\frac{a}{p} + \frac{c}{q}$

$$= \frac{b}{q} + \frac{c}{q} = \frac{(b+c)}{q} = \frac{2q}{q} = 2$$

23. $S_n = a.2^n - b$

$\therefore\ S_{n-1} = a.2^{n-1} - b$

$\therefore\ T_n = S_n - S_{n-1} \quad (n > 1)$

$T_n = a(2^n - 2^{n-1})$

$\therefore\ T_n = 2^{n-1}.a$

Clearly $S_1 = 2a - b$

Series is : 2a–b, 2a, 4a, 8a ....

So it is GP from T_2 onwards

24. Let a, b are roots of a Quadratic eqn. such that

AM : $\frac{a+b}{2} = A \Rightarrow a + b = 2A$

& GM : $\sqrt{ab} = G \Rightarrow ab = G^2$

$\therefore$ eqn. is : $x^2 - Sx + P = 0$

$x^2 - (a + b)x + (ab) = 0$

$\Rightarrow\ x^2 - 2Ax + G^2 = 0$

25. $\therefore$ Roots a, b = $\frac{2A \pm \sqrt{4A^2 - 4G^2}}{2}$

$\therefore\ a,b = A \pm \sqrt{A^2 - G^2}$

Hence both are true

& Statement-II is correct explanation

26. We know geometric mean of 3 numbers x_1, x_2, x_3 is $\sqrt[3]{x_1.x_2.x_3}$

Given, if observations are x_1, x_2, 12; G.M. is 6

$\Rightarrow\ \sqrt[3]{x_1.x_2.12} = 6$

$\Rightarrow\ x_1 \times x_2 \times 12 = 6^3 = 216$

$\Rightarrow\ x_1 \times x_2 = \frac{216}{12} = 18$ ...(i)

Also, given that actual number is 8.

$\therefore$ Actual G.M. = $\sqrt[3]{x_1.x_2.8} = \sqrt[3]{18\times 8}$ (from (i))

$= \sqrt[3]{18\times 2\times 2\times 2} = 2.\sqrt[3]{18}$

27. Let 'a' and 'b' two numbers.

A.M. = $\frac{a+b}{2}$ and G.M. = $\sqrt{ab}$

According to the question,

A : G = m : n

$\Rightarrow\ \frac{a+b}{2\sqrt{ab}} = \frac{m}{n} \Rightarrow \frac{(a+b)^2}{4ab} = \frac{m^2}{n^2}$ ...(i)

and $\frac{(a+b)^2 - 4ab}{4ab} = \frac{m^2 - n^2}{n^2}$

$\Rightarrow\ \frac{(a-b)^2}{4ab} = \frac{m^2 - n^2}{n^2}$ ...(ii)

Since, on dividing Equation (i) and (ii), we get

$\frac{(a+b)^2}{(a-b)^2} = \frac{m^2}{m^2 - n^2} \Rightarrow \frac{(a+b)}{(a-b)} = \frac{m}{\sqrt{m^2 - n^2}}$

$\Rightarrow\ \frac{(a+b)+(a-b)}{(a+b)-(a-b)} = \frac{m+\sqrt{m^2-n^2}}{m-\sqrt{m^2-n^2}}$

(Using componendo dividendo rule)

$\Rightarrow\ \frac{2a}{2b} = \frac{a}{b} = \frac{m+\sqrt{m^2-n^2}}{m-\sqrt{m^2-n^2}}$

28. Let a, ar and ar^2 be three positive terms of G.P.

According to question,

$a = \frac{1}{3}(ar + ar^2)$

$\Rightarrow\ 3 = r + r^2$

$\Rightarrow\ r^2 + r - 3 = 0$

$\Rightarrow\ r = \frac{-1 \pm \sqrt{1+4\times 3}}{2}$

$\Rightarrow\ r = \frac{-1 \pm \sqrt{13}}{2} = \frac{\sqrt{13}-1}{2}, -\left(\frac{1+\sqrt{13}}{2}\right)$

Since, r can not be negative.

$\therefore\ r = \frac{\sqrt{13}-1}{2}$

29. $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+....$

$= \left(1-\frac{1}{2}\right)+\left(1-\frac{1}{4}\right)+\left(1-\frac{1}{8}\right)+\left(1-\frac{1}{16}\right)+....$

$= (1+1+1+ \dots n) - \left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+....\right)$

$= n - \left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+....\right)$

$= n - \dfrac{\frac{1}{2}\left(1-\frac{1}{2^n}\right)}{1-\frac{1}{2}} \quad \left(\because \text{G.P. } a=\frac{1}{2}, r=\frac{1}{2}\right)$

$= n - \dfrac{\frac{1}{2}\left(1-\frac{1}{2^n}\right)}{\frac{1}{2}}$

$= n - 1 + 2^{-n}$

$= 2^{-n} + n - 1$

30. $S_n = np + \frac{n(n-1)Q}{2}$

We know, $T_1 = S_1$ and $T_2 = S_2 - S_1$

Common different (d) $= T_2 - T_1$

$\therefore \quad S_1 = (1)P + \frac{1(1-1)Q}{2} = P + 0 = P$

$S_2 = (2)P + \frac{2(2-1)Q}{2} = 2P + \frac{2Q}{2} = 2P + Q$

$\therefore \quad T_1 = P;\ T_2 = 2P + Q - P = P + Q$

$\therefore \quad$ Common difference (d) $= T_2 - T_1$

$= P + Q - P = Q.$